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# Universal Parameterization of Absorption Cross Sections

## Light Systems

R. K. Tripathi Langley Research Center, Hampton, Virginia

Francis A. Cucinotta Johnson Space Center, Houston, Texas

John W. Wilson Langley Research Center, Hampton, Virginia

National Aeronautics and Space Administration

Langley Research Center Hampton, Virginia 23681-2199

# Available from: NASA Center for AeroSpace Information (CASI) National Technical Information Service (NTIS) 5285 Port Royal Road Springfield, VA 22161-2171 (703) 605-6000 7121 Standard Drive

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(301) 621-0390

### **Symbols**

$A_P$	mass number of projectile nucleus
$A_T$	mass number of target nucleus
В	Coulomb barrier
$C_E$	function related to transparency and Pauli blocking
D	parameter related to density of colliding system
Ε	colliding energy, A MeV
$E_{\rm cm}$	center of mass energy of colliding system, A MeV
G	high-energy parameter for $\alpha + X$ system
R	energy-dependent radius of colliding system
$R_c$	system-dependent Coulomb multiplier
$r_P$	hard sphere radius of projectile nucleus
$r_{ m rms}$	root-mean-square radius
$r_T$	hard sphere radius of target nucleus
$r_0$	constant related to radius of a nucleus
S	mass asymmetry term
$S_L$	function used in optical model multiplier
$T_{I}$	parameter related to surface of colliding system
$X_m$	optical model multiplier
$X_1$	target-dependent function used in optical model multiplier
$Z_P$	charge number of projectile nucleus
$Z_T$	charge number of target nucleus
δ	energy-dependent or energy-independent parameter
$\delta_E$	energy-dependent function
$\sigma_{el}$	elastic cross section

- $\sigma_R$  total reaction cross section
- $\sigma_T$  total cross section

#### **Abstract**

Our prior nuclear absorption cross sections model (NASA Technical Paper 3621) is extended for light systems ( $A \le 4$ ) where either both projectile and target are light particles or one is a light particle and the other is a medium or heavy nucleus. The agreement with experiment is excellent for these cases as well. Present work in combination with our original model provides a comprehensive picture of absorption cross sections for light, medium, and heavy systems, a very valuable input for radiation protection studies.

#### Introduction

The transportation of energetic ions in bulk matter is of direct interest in several areas (refs. 1 and 2), including shielding against ions originating from either space radiations or terrestrial accelerators, cosmic ray propagation studies in galactic medium, or radiobiological effects resulting from the work place or clinical exposures. For carcinogenesis, terrestrial radiation therapy, and radiobiological research, knowledge of the beam composition and interactions is necessary to properly evaluate the effects on human and animal tissues. For the proper assessment to radiation exposures, both reliable transport codes and accurate input parameters are needed.

One such important input is the total reaction cross section  $\sigma_R$ , defined as the total  $\sigma_T$  minus the elastic cross sections  $\sigma_{el}$ , for two colliding ions:

$$\sigma_R = \sigma_T - \sigma_{\rm el} \tag{1}$$

A model has been developed for absorption cross sections (refs. 3 to 6) that gives very reliable results for the entire energy range from a few A MeV to a few A GeV. It is gratifying to note that several agencies and institutions have adopted the model and are using it with success in their programs. The present work extends the model to lighter systems, where either or both projectile and target are light particles. The details of our previous model are discussed elsewhere. (See refs. 3 to 6.) The main features of the formalism are reproduced for completeness and to put the light systems in proper context.

#### **Model Description**

Most of the empirical models approximate total reaction cross section of the Bradt-Peters form:

$$\sigma_R = \pi r_0^2 \left( A_P^{1/3} + A_T^{1/3} - \delta \right)^2 \tag{2}$$

where  $r_0$  is a constant related to the radius of a colliding ion,  $\delta$  is either a constant or an energy-dependent parameter, and  $A_P$  and  $A_T$  are the projectile and target mass numbers, respectively. This form of parameterization works nicely for higher energies. However, at lower energies for charged ions, Coulomb interaction becomes important and modifies reaction cross sections significantly. For the neutron-nucleus collisions, there is no Coulomb interaction, but the total reaction cross section for these collisions is modified by the strength of the imaginary part of the optical potential at the surface, which was incorporated by introducing a low-energy multiplier  $X_m$  that accounts for the strength of the optical model interaction. Because the same form of parameterization is used for the neutron-nucleus case as well (refs. 4 and 6) which helped to provide a unified, consistent, and accurate picture of the total reaction cross sections for any system of colliding nuclei for the entire energy range—the absorption cross sections for light systems are incorporated in the same formalism also. Note that strong absorption models suggest energy dependence of the interaction radius. Incorporating these effects, and other effects discussed later, the following form for the reaction cross section is used as before:

$$\sigma_R = \pi r_0^2 \left( A_P^{1/3} + A_T^{1/3} + \delta_E \right)^2 \left( 1 - R_c \frac{B}{E_{\rm cm}} \right) X_m$$
 (3)

where  $r_0 = 1.1$  fm and  $E_{\rm cm}$  is the colliding system center of mass energy in A MeV. The second to last term on the right-hand side is the Coulomb interaction term which modifies the cross section at lower energies and becomes less important as the energy increases (typically after several tens of A MeV). The Coulomb multiplier  $R_c$  is needed in order to have the same formalism for the absorption cross sections for

light, medium, and heavy systems and for reasons discussed later. In equation (3), B is the energy-dependent Coulomb interaction barrier (right-hand factor in eq. (3)), and is given by

$$B = \frac{1.44Z_p Z_T}{R} \tag{4}$$

where  $Z_P$  and  $Z_T$  are atomic numbers of the projectile and target, respectively, and R, the radius for evaluating the Coulomb barrier height, is

$$R = r_P + r_T + \frac{1.2 \left( A_P^{1/3} + A_T^{1/3} \right)}{E_{\rm cm}^{1/3}}$$
 (5)

where  $r_i$  is equivalent hard sphere radius and is related to the  $r_{{\rm rms},i}$  radius by

$$r_i = 1.29 r_{\text{rms},i} \tag{6}$$

with i = P,T. The computer routine to calculate the radius of a nucleus is given in reference 7.

Energy dependence in the reaction cross section at intermediate and higher energies is mainly because of two effects—transparency and Pauli blocking; this is taken into account in  $\delta_E$ , which is

$$\delta_E = 1.85S + \frac{0.16S}{E_{\rm cm}^{1/3}} - C_E + \frac{0.91(A_T - 2Z_T)Z_P}{A_T A_P}$$
 (7)

where S is the mass asymmetry term, defined as

$$S = \frac{A_P^{1/3} A_T^{1/3}}{A_P^{1/3} + A_T^{1/3}} \tag{8}$$

and is related to the volume overlap of the collision system. The last term on the right-hand side of equation (7) accounts for the isotope dependence of the reaction cross section. The term  $C_E$  is related to the transparency and Pauli blocking and is given by

$$C_E = D \left[ 1 - \exp\left(-\frac{E}{T_1}\right) \right] - 0.292 \, \exp\left(-\frac{E}{792}\right)$$

$$\times \cos 0.229 E^{0.453} \tag{9}$$

where the collision kinetic energy E is in A MeV. Here D is related to the density dependence of the colliding system and can be nicely related to the densities of the colliding systems for medium and heavier systems (refs. 3 to 6). This in effect simulates the modifications of the reaction cross sections due to Pauli blocking. Equations (1) to (9) summarize our original model. For systems discussed in our previous work  $T_1 = 40$  in equation (9) gave very good results. For light systems studied here, where both projectile and target are light systems, there is a significant amount of surface in both the projectile and target nuclei and each system behaves somewhat different from the other. The best values of parameter D and  $T_1$  in equation (9) for the cases studied here are as follows:

n(p) + X systems:

$$T_1 = 18(23)$$

$$D = 1.85 + \frac{0.16}{1 + \exp[(500 - E)/200]}$$
(10)

d + X systems:

$$T_1 = 23$$

$$D = 1.65 + \frac{0.1}{1 + \exp[(500 - E)/200]}$$
(11)

<sup>3</sup>He + X systems:

$$T_1 = 40 D = 1.55$$
 (12)

<sup>4</sup>He + X systems:

$$D = 2.77 - (8.0 \times 10^{-3}) A_T + (1.8 \times 10^{-5}) A_T^2 - \frac{0.8}{1 + \exp[(250 - E)/G]}$$
(13)

Table 1 gives the values of the parameters  $T_1$  and G for alpha-nucleus systems.

For medium and heavy systems, D can be expressed in a very simple way in terms of the densities of the colliding nuclei. (See refs. 3 to 6.) Interesting physics is associated with constant D. The parameter D in effect simulates the modifications of

the reaction cross sections due to Pauli blocking. This effect is new and has not been taken into account in other empirical calculations. The introduction of the parameter D and its association with the physical phenomenon of Pauli blocking helps present a universal picture of the reaction cross sections. At lower energies (below several tens of A MeV) where the overlap of interacting nuclei is small (and where Coulomb interaction and imaginary part of the optical potential modify the reaction cross sections significantly), the modifications of the cross sections due to Pauli blocking are small and gradually play an increasing role as the energy increases because this leads to higher densities where Pauli blocking gets increasingly important.

This method of calculation of Coulomb energy does provide a unified picture of reaction cross sections for any system of colliding nuclei. For light systems, equation (5) overestimates the interaction distance and consequently equation (4) underestimates the Coulomb energy effect. In order to compensate for this effect and still maintain the same formalism for light, medium, and heavy systems, there was a need to introduce a Coulomb multiplier parameter  $R_c$  in equation (3). Table 2 gives the values of  $R_c$  for the cases studied here. The optical model multiplier as introduced in references 4 and 6 is given by

$$X_m = 1 - X_1 \exp\left(-\frac{E}{X_1 S_L}\right) \tag{14}$$

with

$$X_1 = 2.83 - (3.1 \times 10^{-2})A_T + (1.7 \times 10^{-4})A_T^2$$
 (15)

For the n +  ${}^{4}$ He system,  $X_1 = 5.2$  gives better agreement with experiment. The function  $S_L$  for light systems as used here is

$$S_L = 1.2 + 1.6 \left[ 1 - \exp\left(-\frac{E}{15}\right) \right]$$
 (16)

#### Results

Figures 1 to 20 show the plots of available results for neutron-nucleus, proton-nucleus, deuteron-nucleus, helium 3-nucleus, and alpha-nucleus systems. The data in figures 1 and 2 are from reference 8, and data in figure 3 have been taken from references 8 and 9. For figure 4, data have been taken from references 9 to 11. An extensive data set exists for p + <sup>4</sup>He

collisions (fig. 5), and data have been taken from references 8, 9, and 11 to 16. Data for figures 6 and 7 have mainly been collected from the compilation of reference 9, and those of figures 8 to 10 are from reference 12. Not much data are available for <sup>3</sup>He-nucleus collisions and the data have been taken from reference 17 for figures 11 to 13. For <sup>4</sup>He + <sup>4</sup>He (fig. 14), data have been taken from references 12, 13, and 17. For figures 15, data have been taken from reference 17. For figures 16 to 18, data have been taken from reference 18, and those of figures 19 and 20 have been taken from reference 19.

#### **Concluding Remarks**

The agreement of our results with experiments for light systems is excellent for the entire energy range from a few A MeV to a few A GeV and is of the same quality as that of our previous work. Present work in combination with our original model provides a comprehensive picture of absorption cross sections for light, medium, and heavy systems. We are not aware of any published or reported model which gives as good agreement for absorption cross sections for light, medium, and heavy systems for the entire energy range as found here.

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Table 1. Parameters for  $\alpha$  + X Systems

System	$T_1$	G
General setting	40	75
$\alpha + \alpha$	40	300
α + Be	25	300
$\alpha + N$	40	500
α + Al	25	300
α + Fe	40	300

Table 2. Coulomb Multiplier for Light Systems

System	$R_{\rm c}$
p + d	13.5
$p + {}^{3}He$	21
p + <sup>4</sup> He	27
p + Li	2.2
d + d	13.5
d + <sup>4</sup> He	13.5
d + C	6.0
<sup>4</sup> He + Ta	0.6
<sup>4</sup> He + Au	0.6

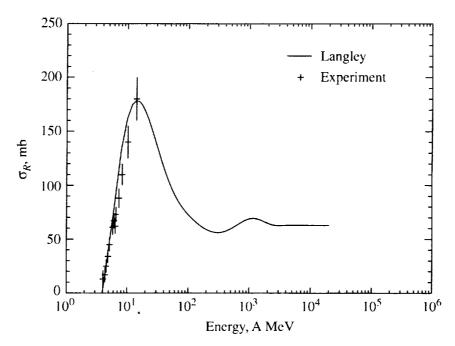


Figure 1. Reaction cross sections as a function of energy for n + d collisions.

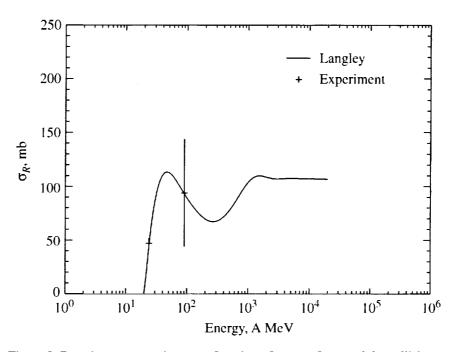


Figure 2. Reaction cross sections as a function of energy for n + alpha collisions.

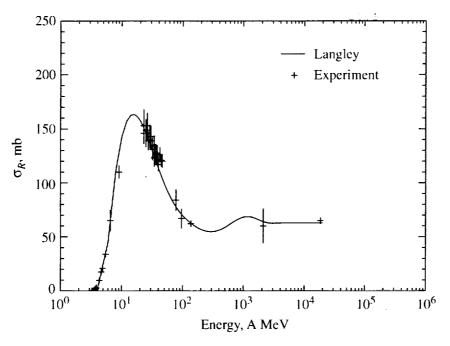


Figure 3. Reaction cross sections as a function of energy for p + d collisions.

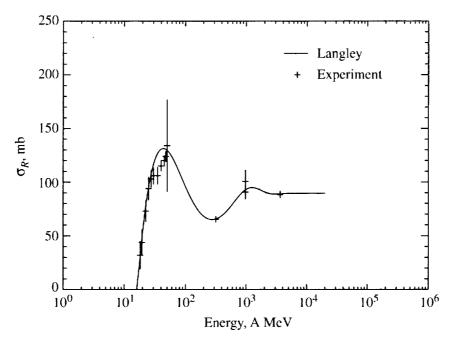


Figure 4. Reaction cross sections as a function of energy for p + helium 3 collisions.

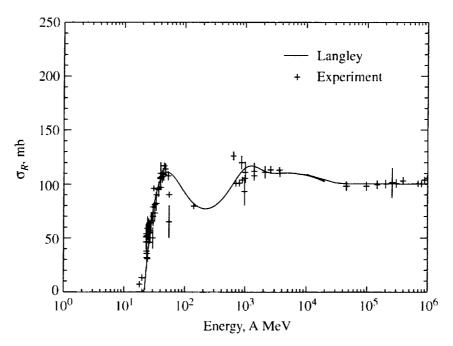


Figure 5. Reaction cross sections as a function of energy for p + alpha collisions.

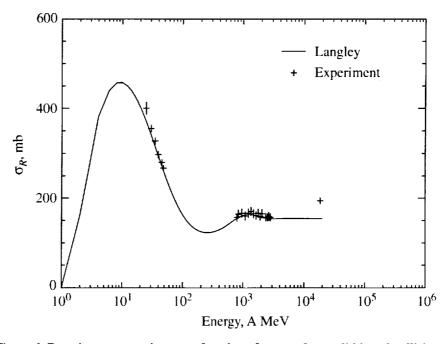


Figure 6. Reaction cross sections as a function of energy for p + lithium 6 collisions.

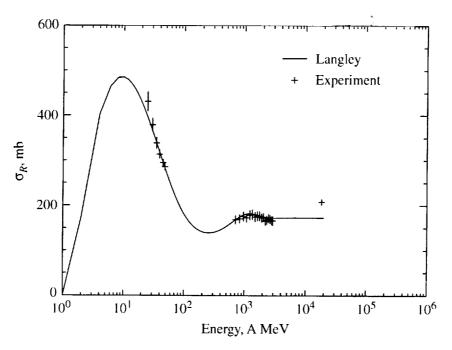


Figure 7. Reaction cross sections as a function of energy for p + lithium 7 collisions.

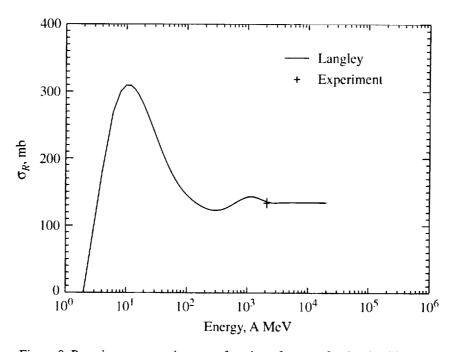


Figure 8. Reaction cross sections as a function of energy for d + d collisions.

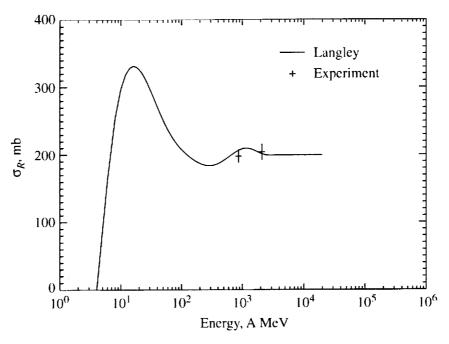


Figure 9. Reaction cross sections as a function of energy for d + alpha collisions.

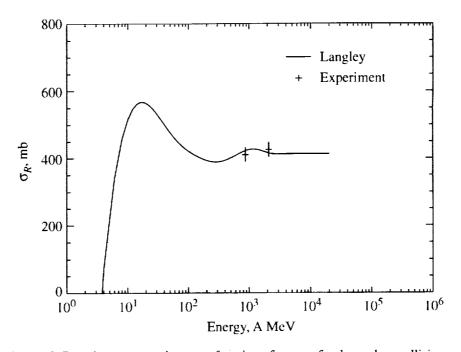


Figure 10. Reaction cross sections as a function of energy for d + carbon collisions.

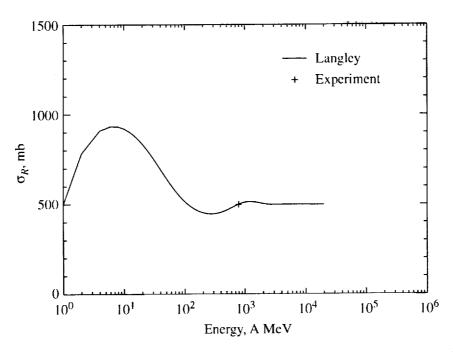


Figure 11. Reaction cross sections as a function of energy for helium 3 + beryllium collisions.

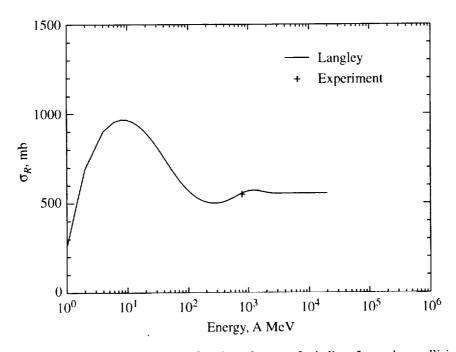


Figure 12. Reaction cross sections as a function of energy for helium 3 + carbon collisions.

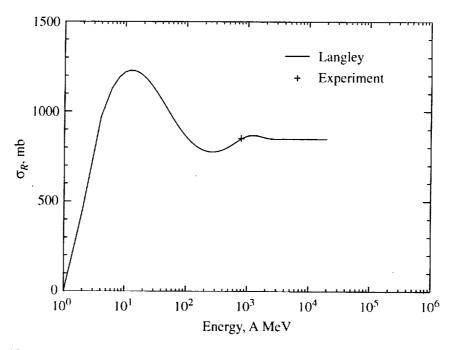


Figure 13. Reaction cross sections as a function of energy for helium 3 + aluminum collisions.

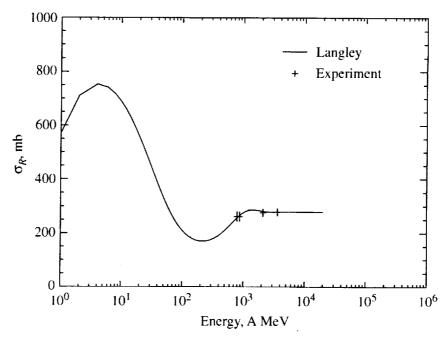


Figure 14. Reaction cross sections as a function of energy for alpha + alpha collisions.

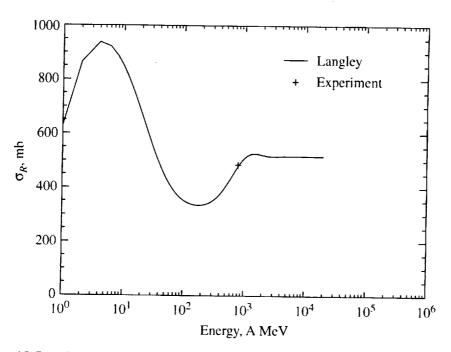


Figure 15. Reaction cross sections as a function of energy for alpha + beryllium collisions.

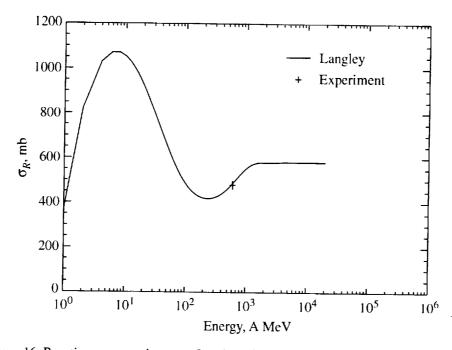


Figure 16. Reaction cross sections as a function of energy for alpha + nitrogen collisions.

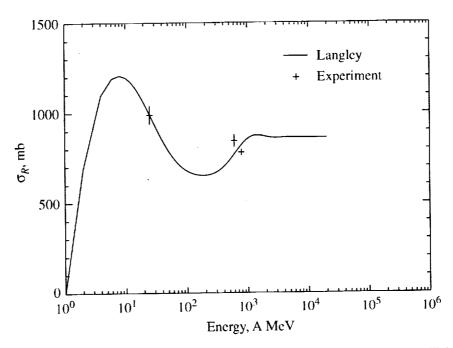


Figure 17. Reaction cross sections as a function of energy for alpha + aluminum collisions.

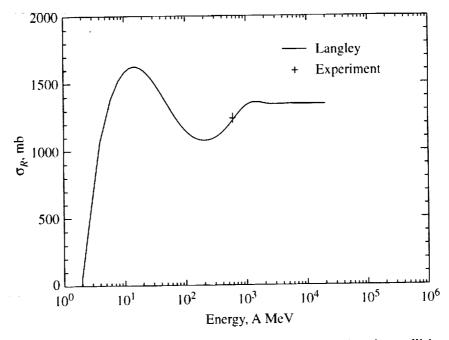


Figure 18. Reaction cross sections as a function of energy for alpha + iron collisions.

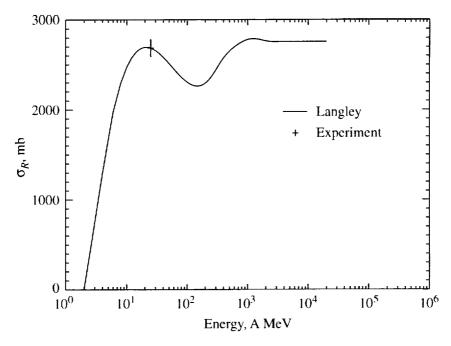


Figure 19. Reaction cross sections as a function of energy for alpha + tantalum collisions.

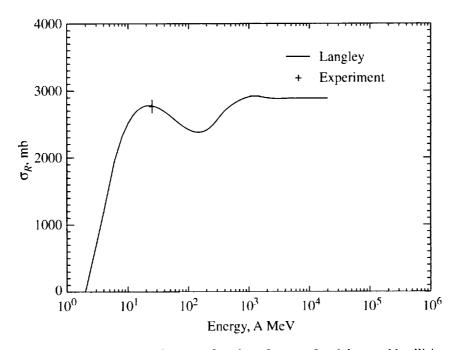


Figure 20. Reaction cross sections as a function of energy for alpha + gold collisions.

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		ASA Technical Paper	3621) is extended for light systems
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